Applying the Inverse Average Magnitude Squared Coherence Index for Determining Order-Chaos Transition in a System Governed by Hénon Mapping Dynamics

Carlos R. Fadragas*

Physics Department, Faculty of Mathematics, Physics and Computer Science, Central University of Las Villas, Santa Clara, Cuba

Rubén Orozco Morales†

Center for Studying Electronics and Information Technologies, Faculty of Electrical Engineering, Central University of Las Villas, Santa Clara, Cuba

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Abstract

The quantitative determination of the order-chaos transition in a nonlinear dynamical system described by Hénon mapping defined as $x[n+1] = 1.0 - A * x[n]^2 + B * y[n], y[n+1] = B * x[n]$, where $B = 0.3$, and A is an adjustable control parameter, was made. This was achieved by applying the Inverse Average Magnitude-Squared Coherence Index (IAMSCI). This method is based on the Welch average periodogram technique and it has the advantage respect to nonlinear dynamical methods that it may be applied to any stationary signal by using discrete Fourier transform (DFT) representation which allows to operate on a short discrete-time series. Its effectiveness was demonstrated by comparing the results obtained by applying IAMSCI

*fadragas@mfc.uclv.edu.cu

†rorozco@fie.uclv.edu.cu

method with those obtained by calculating the largest Lyapounov exponent (LLE), both applied to the same discrete-time series set derived from the Hénon mapping.

I. INTRODUCTION

Nonlinear dynamical methods for analyzing a discrete-time series and for determining order-chaos transition in a dynamical system have been widely used. But it is recognized by several authors that nonlinear dynamical methods used imply a high computational cost. In this sense, therefore, any method that allows to obtain some information about the behavior of a dynamical system and that can operate over a relatively shorter discrete-time series, acquires an important practical value. For complementing those methods of nonlinear dynamical analysis we shall now consider to demonstrate that introducing the concept of the inverse average magnitude-squared coherence index is possible to quantify the order-chaos transition in a system governed by the Hénon mapping dynamics. It is very important when dealing with a chaotic system keeping in mind that local sensitivity to a small error is the hallmark of such a system. A dynamical system such those that are described by the Hénon mapping dynamics produces a discrete-time non-chaotic series when the control parameter A value satisfies the condition $A < 1.0532$. For such a condition a discrete-time series generated by the recursive process does not almost change if the initial condition is modified and there exists phase consistency between the two discrete-time series at a given frequency. However, when the control parameter value is greater than 1.0532, a discrete-time series generated by the Hénon mapping does meaningfully change if the initial condition is slightly modified and phase consistency between the two discrete-time series at a given frequency, in general, must be lost [4]. This effect may be quantified by applying an inverse average magnitude-squared coherence index [5]. The magnitude-squared coherence has been used by several authors for measuring phase constancy between two or more signals at one frequency [11,7,12,2,9,6] In former works [3,4] the DFT representation for qualitatively describing the order-chaos transition in a dynamical system is used. By introducing an inverse average magnitude-squared coherence index (IAMSCI) the order-chaos transition in a dynamical system described by the logistic equation was quantitatively determined [5]. Now we intended, following this idea, to make the description of order-chaos

transition in a dynamical system governed by the Hénon mapping dynamics by applying the Welch average periodogram method for calculating the power spectral density and from this, to introduce the inverse average magnitude-squared coherence index for indicating quantitatively order-chaos transition in the dynamical system indicated. The fundamental advantage of this method upon the others is that it can operate on a relatively shorter discrete-time series. By estimating the largest Lyapounov exponent was able to evaluate those results obtained here applying an inverse average magnitude squared coherence index, defined by averaging MSC estimate over all the frequency.

II. METHODS AND MATERIALS

A. Obtaining the discrete-time series set

The Hénon mapping can be formulated as given by

$$x[n+1] = 1.0 - A * x[n]^2 + B * y[n], \tag{1}$$

$$y[n+1] = B * x[n] \tag{2}$$

where B=0.3, and A is an adjustable control parameter. With the initial condition (x[1],y[1]) specified, a discrete-time series with a given length by a recursive action is obtained, each time. The nature of the behavior of a data sequence obtained can be modified by chosen conveniently a value of the control parameter A. The interval [0.2,1.4] was chosen, and a set of evenly spaced values of A with a step δ=0.01 was considered to obtain a family consisting of 120 discrete-time series, each of them containing $N = 2^{10}$ samples. The plot of the histogram for each discrete-time series allows to have immediately a simple statistical characterization of the discrete-time series. In order to make easier further computations, data points were organized into a rectangular matrix containing 120 columns, being each of them a discrete-time series with the length $N = 1024$ data points.

B. Determining the largest Lyapounov exponent λ_1

A system containing one or more positive exponents is to be defined as chaotic. For calculating the Lyapounov exponents spectrum some algorithms have been developed. One of the most used algorithm is that reported by Wolf et al [13]. From the exponents spectrum, the largest exponent, λ_1, decides the behavior of the dynamical system. For calculating the largest exponent, one can also refer the algorithm proposed by Rosenstein et al [8]. Details of these algorithms can be found in refereed papers. The largest Lyapounov exponent,λ_1, for each time series in the set was determined using a professional software [10]. The selection of the time delay, τ, and the embedding dimension, D_e, required for reconstructing the phase space is part of the problem. The value τ=1 was chosen considering the optimal filling of the phase space method [1]. For embedding dimension the value D_e=3 was chosen.

C. Estimating the IAMSCI using the Welch average periodogram

It is highly meaningful when dealing with a chaotic system keeping in mind that local sensitivity to a small error is the hallmark of such a system. A dynamical system as the one described by the logistic equation produces a discrete-time non-chaotic series when the control parameter A value satisfies the condition $A < 1.0532$. For such a condition a discrete-time series generated by the recursive process does not almost change if the initial condition is modified and there exists phase consistency between the two discrete-time series at a given frequency. However, when the control parameter value is greater than 1.0532, a discrete-time series generated by the Hénon mapping does meaningfully change if the initial condition is slightly modified and phase consistency between the two discrete-time series at a given frequency, in general, must be lost. This effect may be quantified by applying an inverse average magnitude-squared coherence index. In order to apply the Welch average periodogram method for determining the power density spectrum $S_x[k]$ of a discrete-time series one follows the following steps: (a) Decompose the sequence of N (= 1024) data points

in M (= 9) segments, each of one having the same length L (= 256), and which may be overlapped P samples. The usual value $P = 0.625 * L$ was chosen. (b) Calculate the DFT representation by fft algorithm for each segment as given by $X_m[k] = fft(x_m[n])$, where $m = 1, 2, \ldots, M$, and $n = 0, 1, 2, \ldots, L-1$. For each segment a L-fft was calculated. (c) Calculate the periodogram for each segment m as given by

$$P_m[k] = \frac{1}{L}|X_m[k]|^2, \tag{3}$$

where $m = 1, 2, \ldots, M$. (d) Finally, calculate the average periodogram as given by

$$S_x[k] = \frac{1}{M}\sum_{m=1}^{M} P_m[k], \tag{4}$$

where $k = 0, 1, 2, \ldots, L-1$. For a short discrete-time series a rectangular window is recommended. For determining the magnitude-squared coherence sequence the procedure described above is applied to a second signal $y[n]$ obtaining the average periodogram $S_y[k]$. The average cross-periodogram $S_{xy}[k]$ is also calculated. The magnitude-squared coherence sequence was calculated as given by

$$MSC_{xy}[k] = \frac{S_{xy}[k] * S^*_{xy}[k]}{S_x[k] * S_y[k]} \tag{5}$$

The $MSC_{xy}[k]$ sequence describes the phase consistency between the two signals at a given frequency k. The mean value of the sequence, $\langle MSC_{xy}[k]\rangle$, may be used as an index of global coherence between the two signals. When dealing with a chaotic discrete-time series this index must tend toward zero, and when dealing with a deterministic discrete-time series this index must tend toward one [5]. However, it was found that a better index may be defined as given by

$$f = 10 * log_{10}\frac{1}{\langle MSC_{xy}[k]\rangle}. \tag{6}$$

where f is given in decibel. It makes easier to compare results obtained here with those obtained by applying the largest Lyapounov exponent estimate method.

III. RESULTS

Figures, from 1 to 4, show a sample of four representative time series and their corresponding magnitude-squared coherence espectra. In order to give a better view of each time series it was only considered 128 data points of each discrete-time series in its plotting, but it was completely considered when its corresponding magnitude-squared coherence was calculated. In either case, the particular values of A parameter were indicated. Figure 5 depicts the results of plotting the inverse average magnitude-squared coherence ($1/\langle MSC_{xy}[k]\rangle$), calculated for each time series in the set, as a function of the control parameter A. And figure 6 depicts the results of plotting the inverse average magnitude-squared coherence index, f, in decibels, calculated for each time series in the set, as a function of the control parameter A too. Figure 7 shows the plotting of the largest Lyapounov exponent estimate, λ_1, calculated for each time series in the set, as a function of the control parameter A. This reference serves as a control plot for the discussion of the results obtained by applying the inverse average magnitude-squared coherence index method.

IV. DISCUSSION OF THE RESULTS

The application of the DFT representation of a chaotic-type signal gives satisfactory results for determining the order-chaos transition in a system described by the Hénon mapping dynamics [4]. A critical value for control parameter A is reported in literature beyond which a sequence produced by the Hénon mapping exhibits a chaotic behavior. This threshold value approaches $A \cong 1.0532$. On the other hand, figures from 1 to 4 show the time domain representation of some of the time series and their corresponding magnitude-squared coherence spectrum. Comparing figures 1 ($A =$) with 2 ($A =$), 3 ($A =$), and 4 ($A =$), it can be deduced the bifurcation effect and the transition to chaos. Note that in figures 2, 3 and 4 the control parameter satisfies the condition $A > 1.0532$, and it corresponds to a value for a positive largest Lyapounov exponent, as it can be estimated from figure 7. It may

be observe in those cases that the magnitude-squared coherence spectrum exhibits values far away from one for several values of the frequency in the MSC spectrum. It makes the average magnitude-squared coherence to be less than one, corresponding to a situation for which the discrete-time series becomes incoherent indicating in this case a chaotic behavior of the dynamical system. This situation can be observed in figures 5 and 6 where either the quantity $(1/\langle MSC_{xy}[k]\rangle)$ as the inverse average MSC index, f, are greater than 1 and 0, respectively, for $A > 1.0532$. Applying the inverse average magnitude-squared coherence index (IAMSCI) to a discrete-time series in a family derived from an observable in a dynamical system is recommended for quantitatively detecting the order-chaos transition, and this can be added to the metric tools of the nonlinear dynamical analysis for complementing the research of a discrete-time series.

V. CONCLUSIONS

The application of the inverse average magnitude-squared coherence index (IAMSCI), f, to a discrete-time series obtained from the Hénon mapping can be done on a chaotic discrete-time series. This discrete-time series has the property that exhibits a high level of coherence lost, and it can be detected by applying the IAMSCI method. This method does not exert a strong requirement on the length of the experimental data sequence. If one deals with a nonlinear dynamical system which can modify its behavior between order and chaos, a discrete-time series produced by that system reflects this change, and the IAMSCI method applying to the discrete-time series allow to determine this change. Any method that permits some quantitative evaluation about the behavior of a dynamical system and that can work over a relatively short time series acquires a particular importance.

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VI. FIGURES

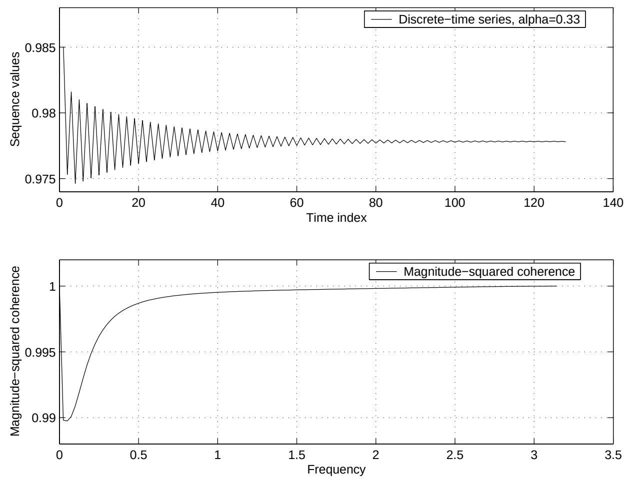


FIG. 1. Discrete-time series and its magnitude-squared coherence, for A=0.33

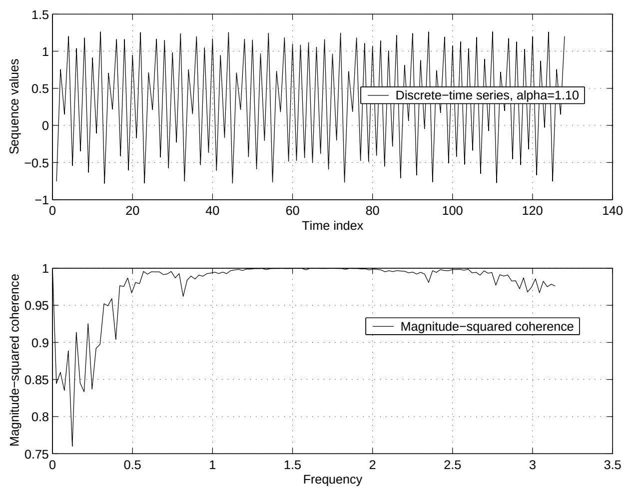


FIG. 2. Discrete-time series and its magnitude-squared coherence, for A=1.10

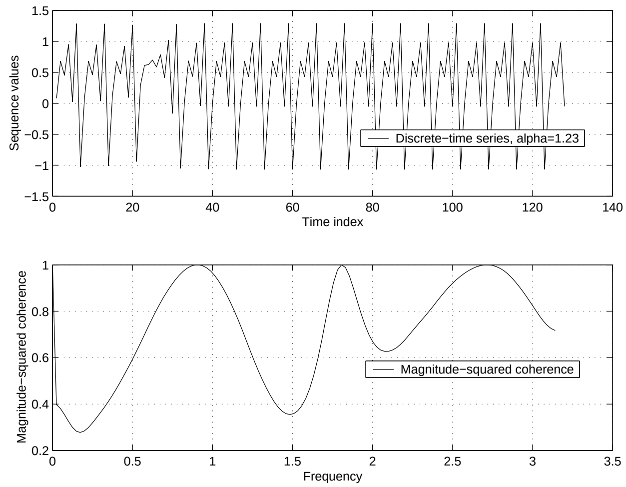


FIG. 3. Discrete-time series and its magnitude-squared coherence, for A=1.23

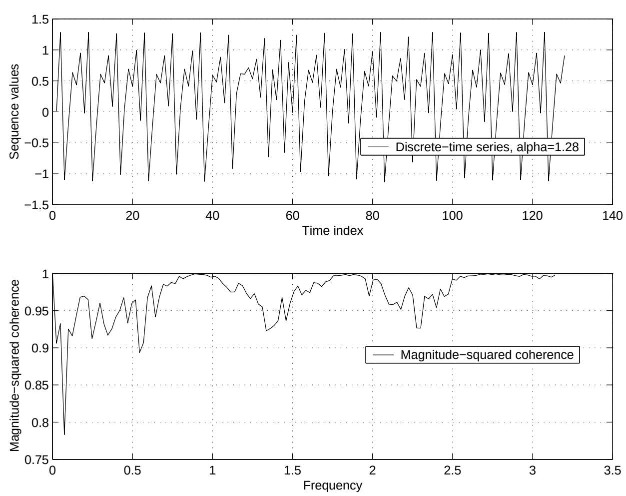


FIG. 4. Discrete-time series and its magnitude-squared coherence, for A=1.28

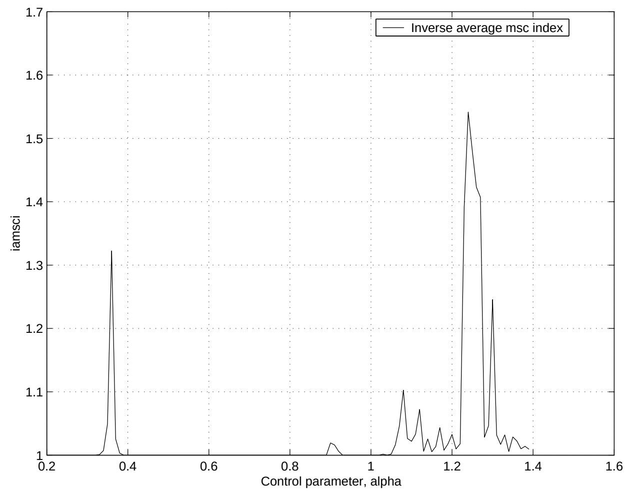


FIG. 5. Inverse average magnitude-squared coherence index

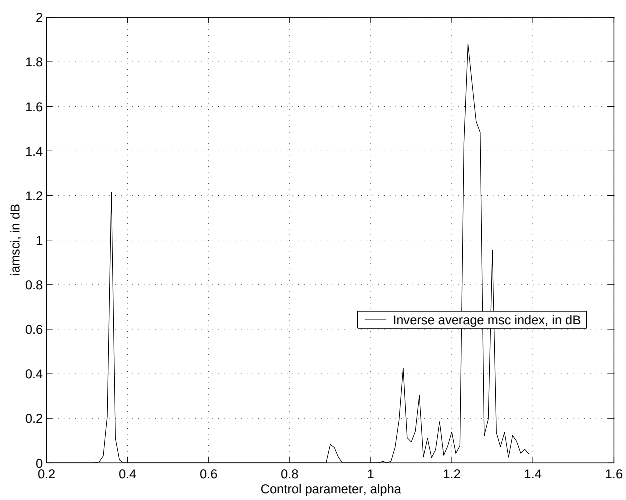


FIG. 6. Inverse average magnitude-squared coherence index, in dB

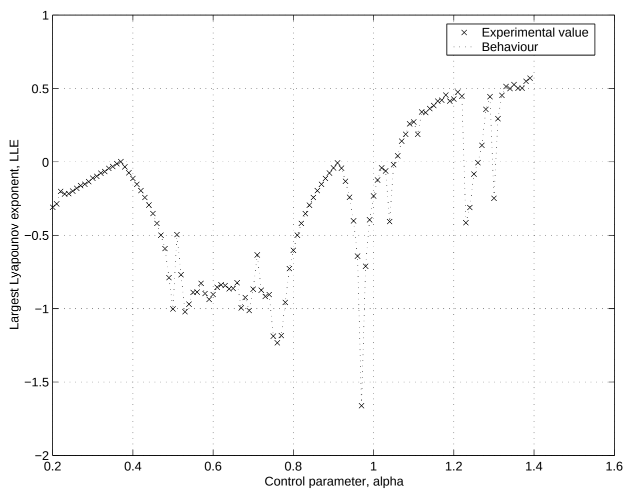
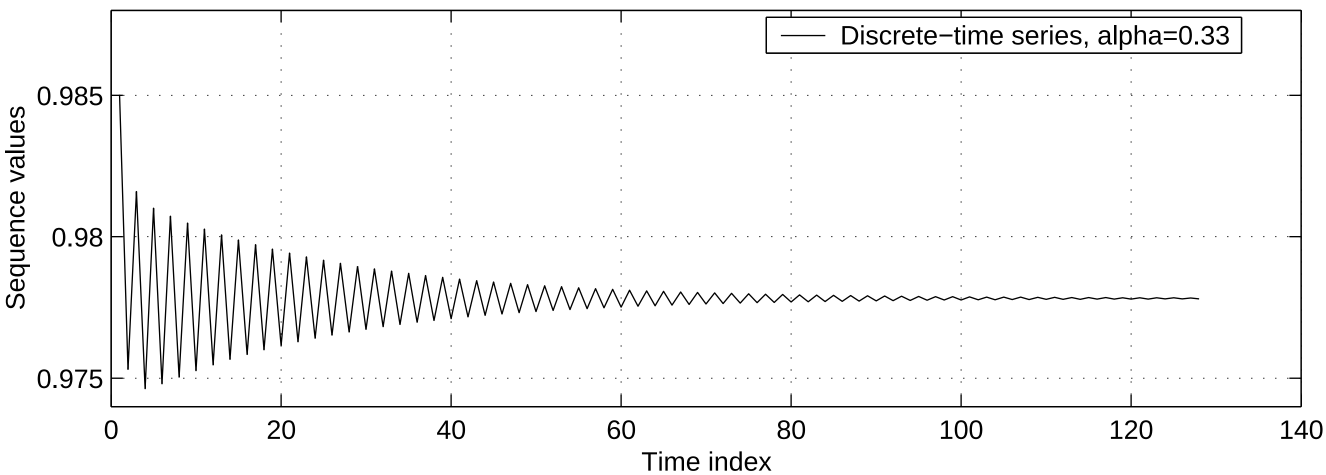
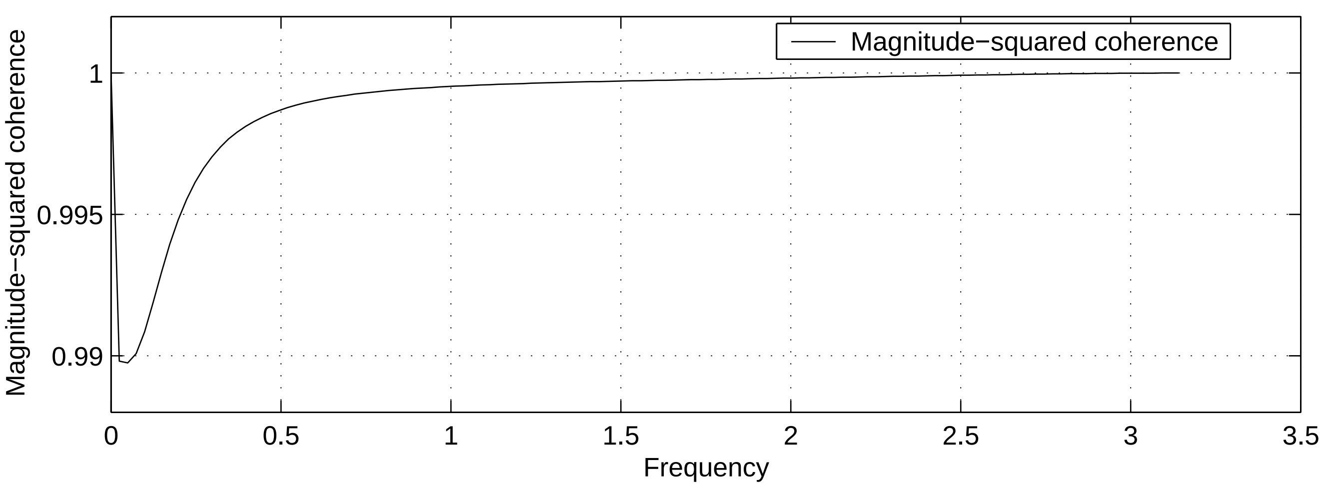


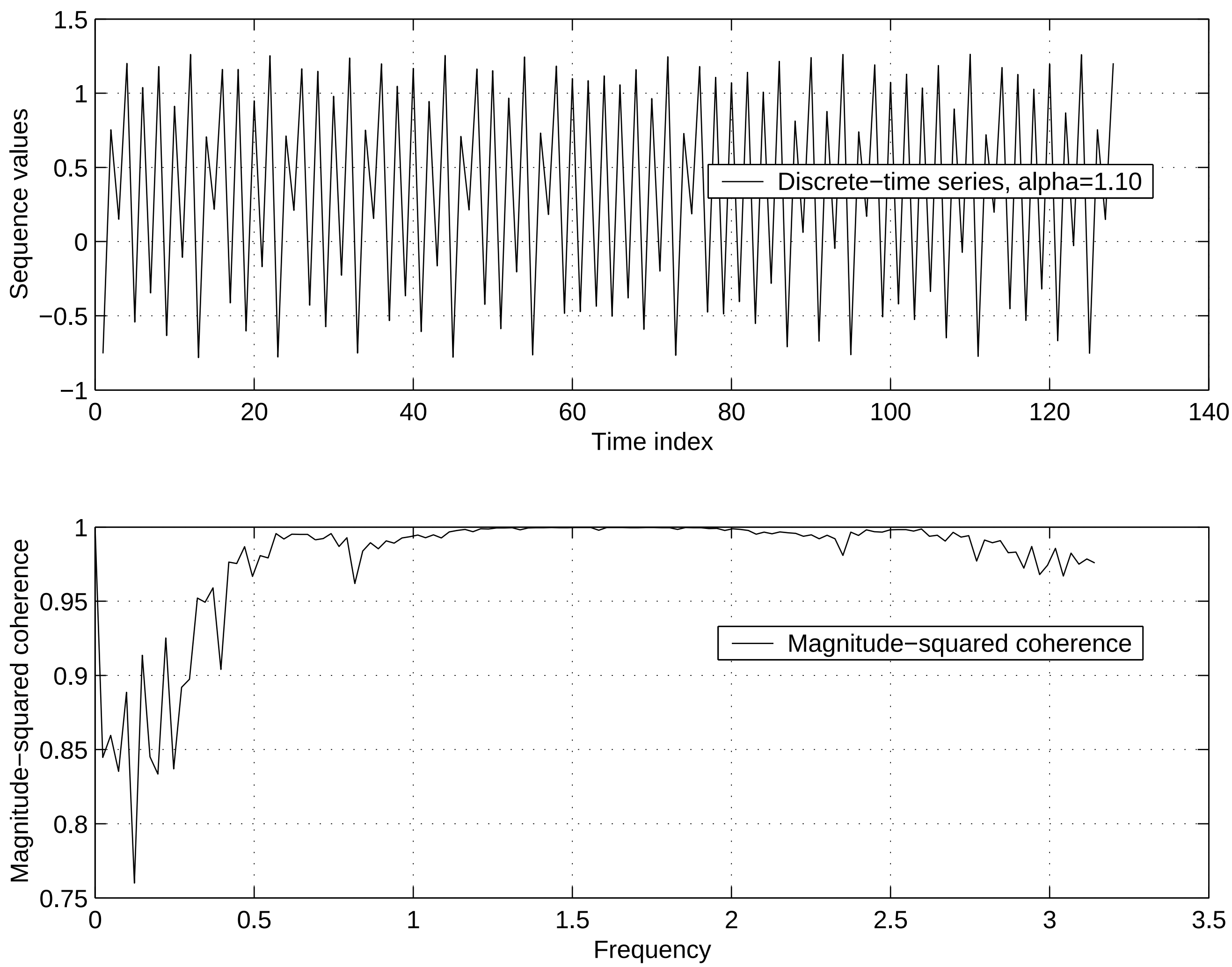
FIG. 7. Largest Lyapounov versus control parameter r



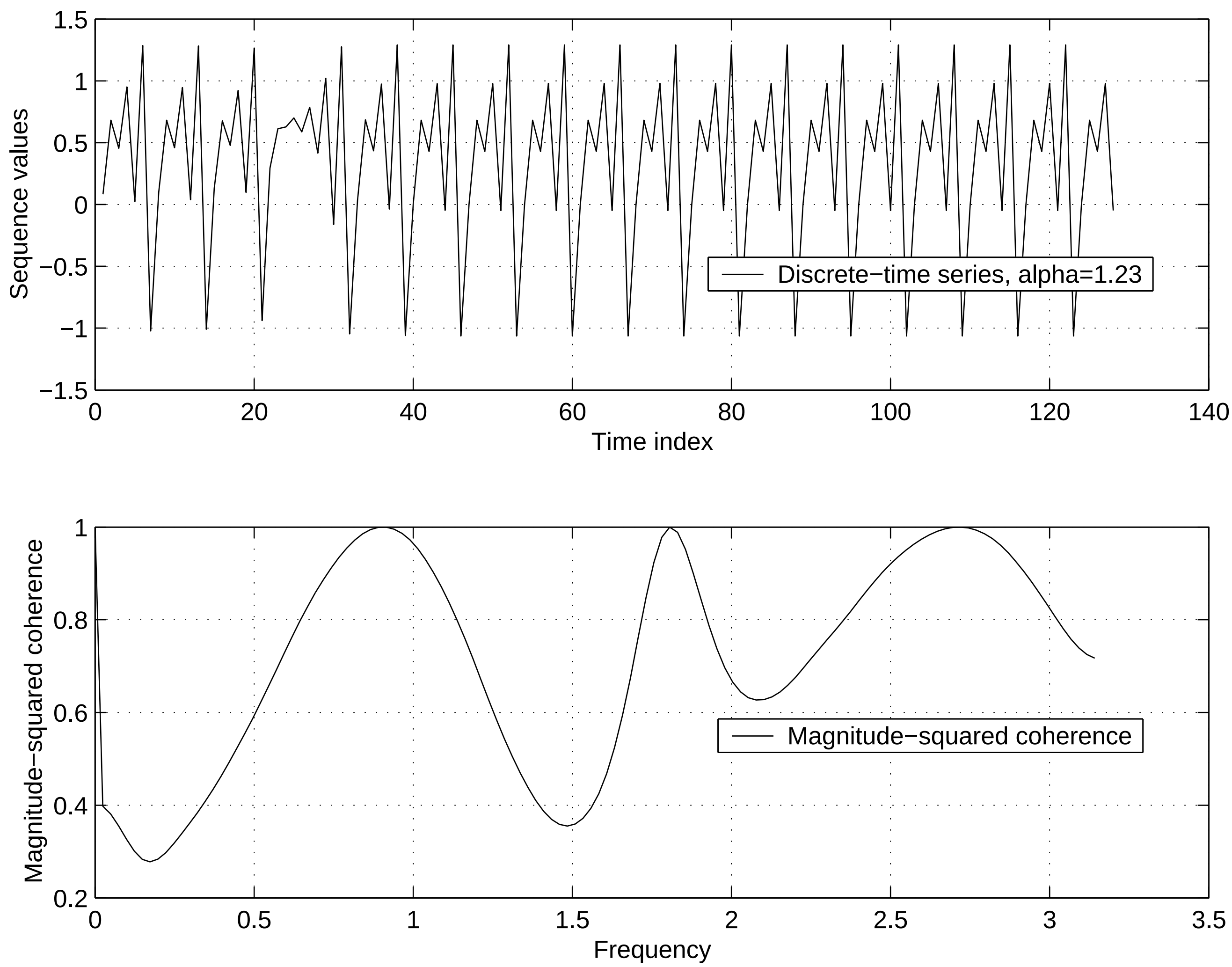
Discrete−time series, alpha=0.33
Sequence values
0.985
0.98
0.975
0
20
40
60
80
100
120
140
Time index

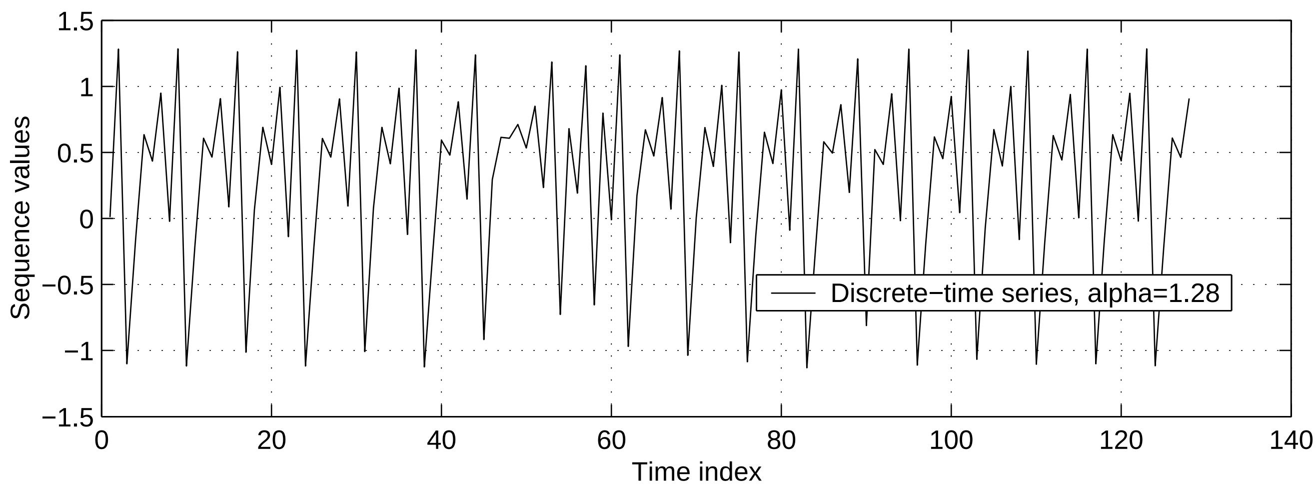


Magnitude−squared coherence
Magnitude−squared coherence
1
0.995
0.99
0
0.5
1
1.5
2
2.5
3
3.5
Frequency

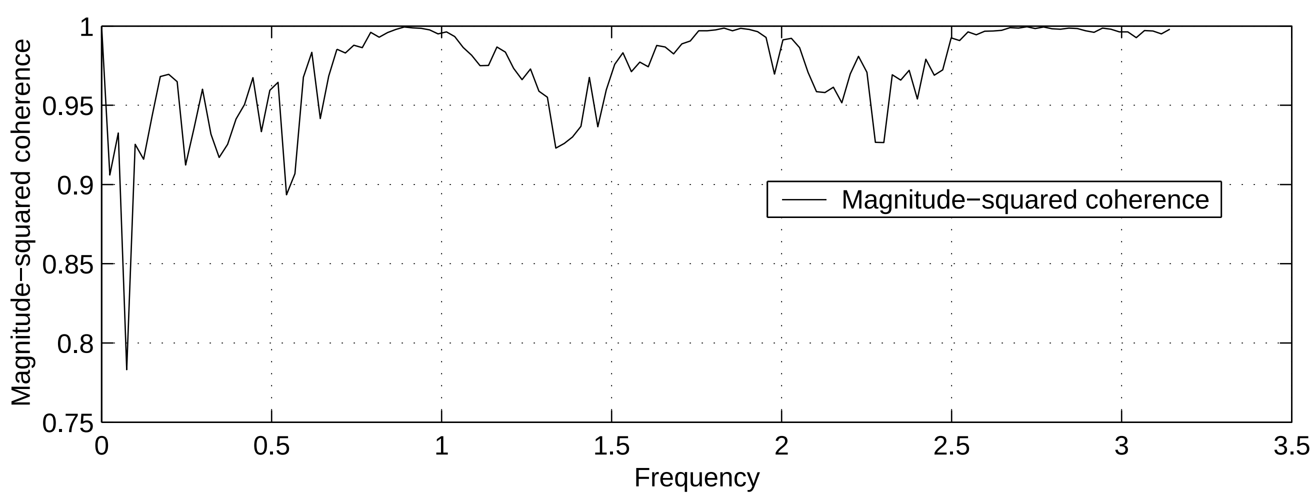


Sequence values
Time index
Discrete−time series, alpha=1.10
Magnitude−squared coherence
Frequency
Magnitude−squared coherence

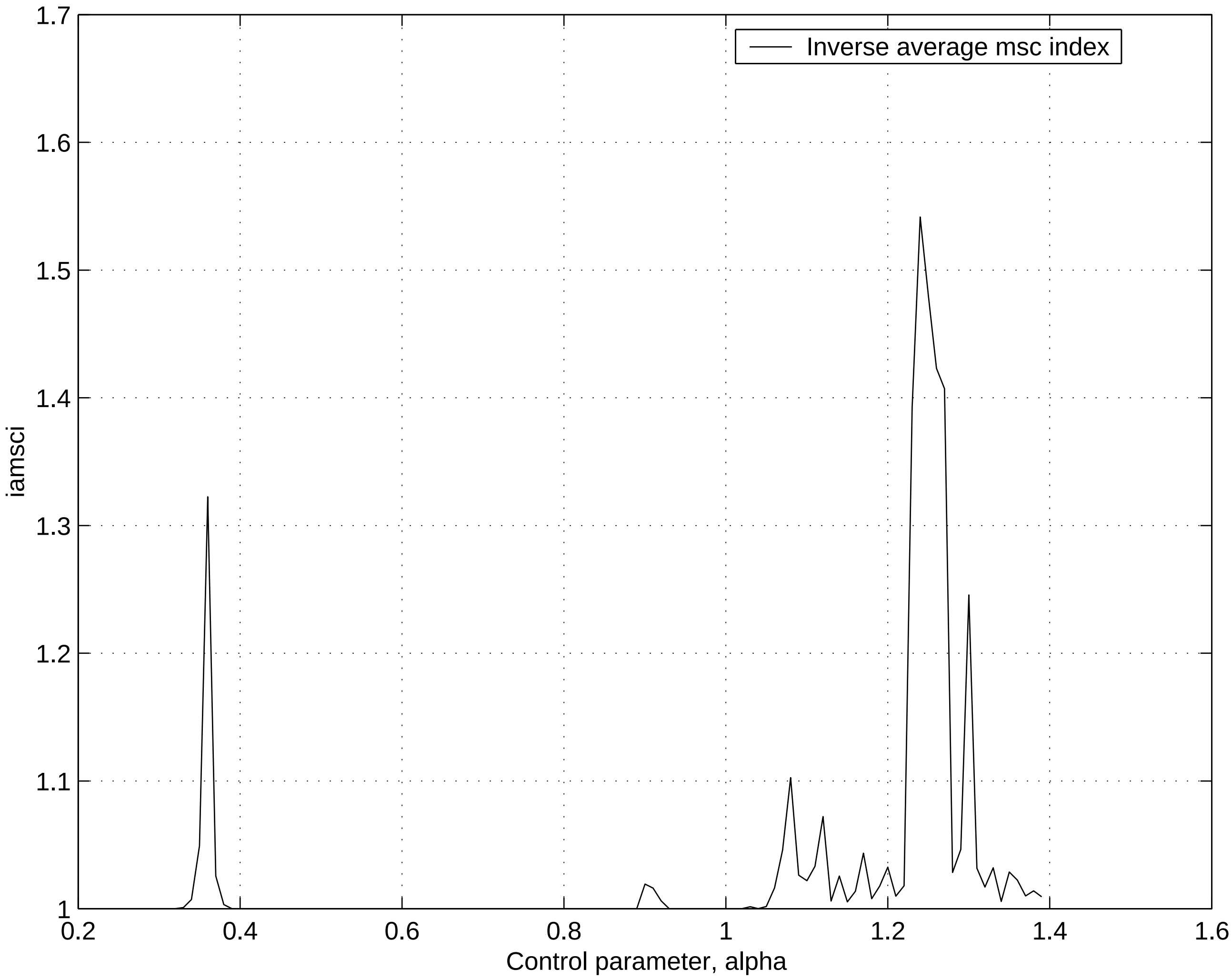

1.5
1
0.5
0
−0.5
−1
−1.5
Sequence values
0
20
40
60
80
100
120
140
Time index
Discrete−time series, alpha=1.23
1
0.8
0.6
0.4
0.2
Magnitude−squared coherence
0
0.5
1
1.5
2
2.5
3
3.5
Frequency
Magnitude−squared coherence

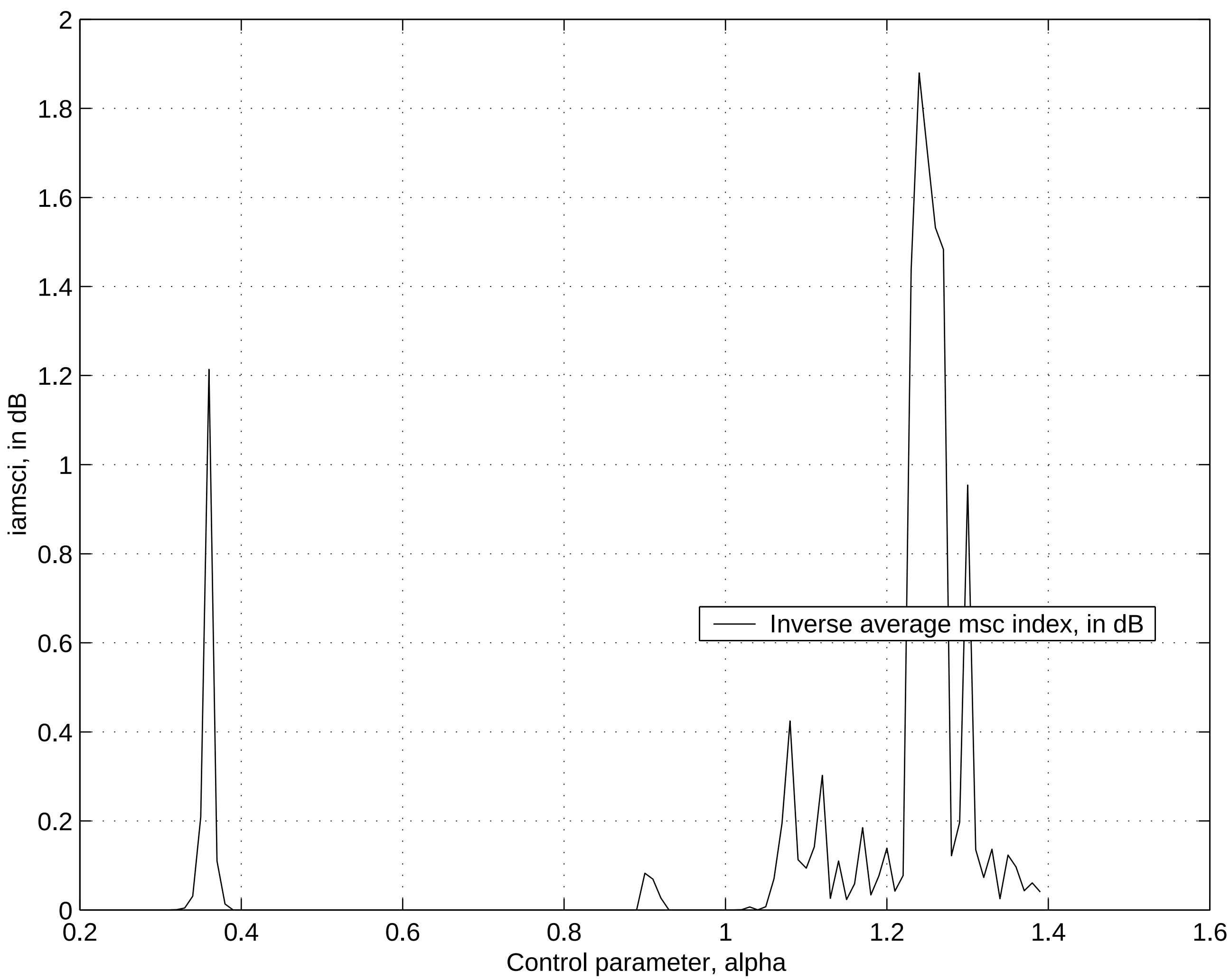


Sequence values
Time index
Discrete−time series, alpha=1.28
1.5
1
0.5
0
−0.5
−1
−1.5
0
20
40
60
80
100
120
140

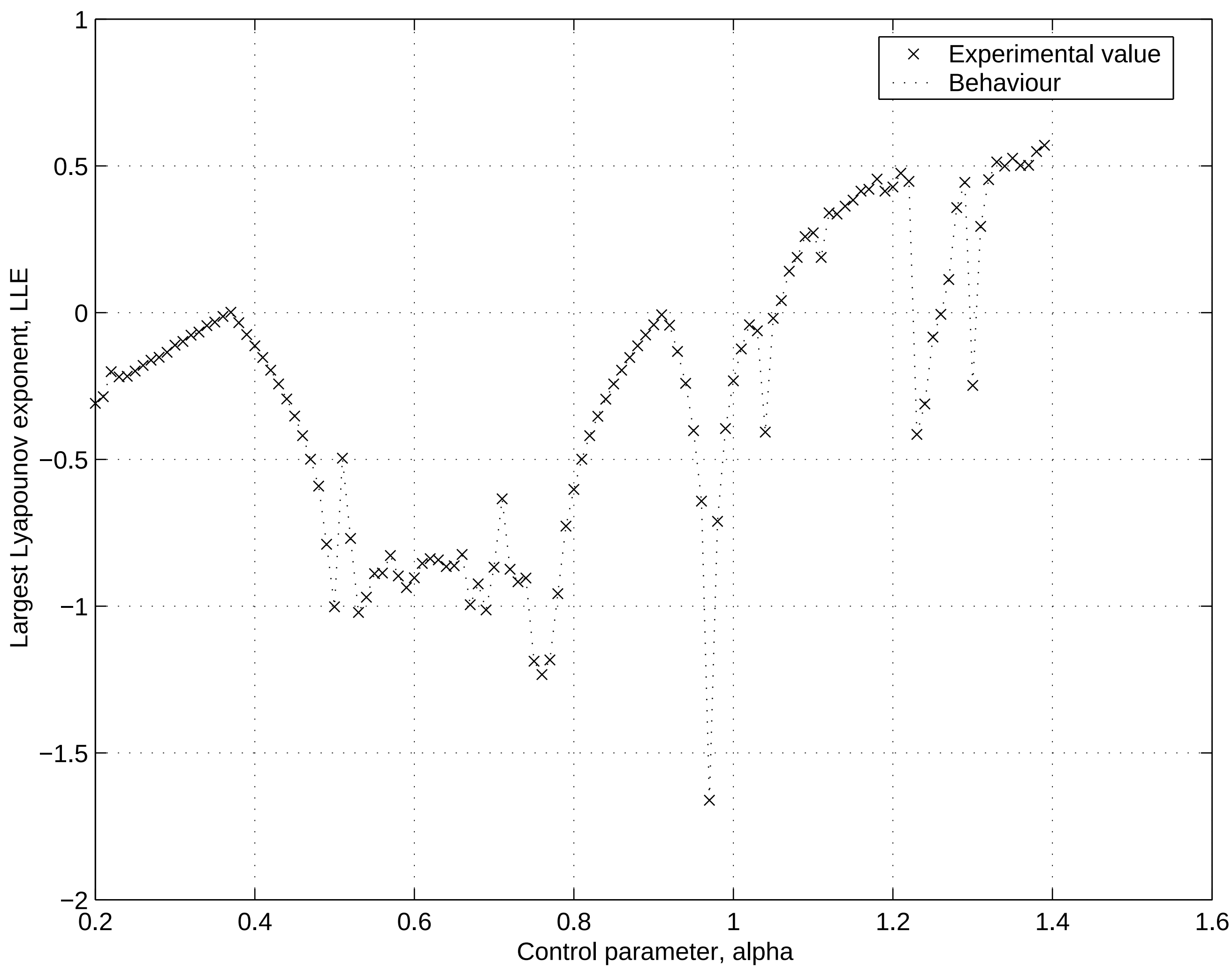


Magnitude−squared coherence
Frequency
Magnitude−squared coherence
1
0.95
0.9
0.85
0.8
0.75
0
0.5
1
1.5
2
2.5
3
3.5


Inverse average msc index
iamsci
Control parameter, alpha
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6



2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
iamsci, in dB
Inverse average msc index, in dB
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Control parameter, alpha



Experimental value
Behaviour
Largest Lyapounov exponent, LLE
Control parameter, alpha
1
0.5
0
−0.5
−1
−1.5
−2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6